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Mars Gravity Field Derived from Viking-1 and Viking-2: The Navigation Result

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A Martian gravity field derived from Viking-1 and Viking-2 Doppler tracking data taken during the ≈1500 km altitude phases is compared to models obtained from Mariner-9. The fields compare favorably in the north, but marked disagreements in the south can only be resolved by proper weighting of a priori Mariner-9 data. Viking orbiters sense the gravity field in the vicinities of 25°N to 55°N whereas Mariner-9 models are most valid in the vicinity of 25°S. The Viking data were reduced to obtain a model of degree and order six in the spherical harmonic expansion of the potential. Since this model is restricted to low-degree terms, the technique used here takes advantage of orbital element variations over periods commensurate with the period of the orbit. This precludes modeling short-period variations near periapsis which usually require higher-degree terms in the potential. The model discussed herein was derived from an ensemble of models obtained over two to four revolutions of Doppler data taken during various synchronous and asynchronous phases of the Viking mission. A priori Mariner-9 fields were also included in the ensemble in an attempt to apply the requisite constraint on the gravity in the south. Short-arc gravity analyses of the low-altitude phases (≈300 km for Viking-1 and ≈800 km for Viking-2) indicate that ensemble sixth degree and order models degrade the gravity field globally; thus, future data reduction incorporating the low-altitude Viking orbits will require higher degree and order models.

Introduction

A MARTIAN gravity field has been derived from Viking-1 and Viking-2 Doppler tracking data taken during orbit phases characterized by 1500 km subperiapse altitudes. This gravity model resulted from efforts of the Viking Navigation Team to periodically upgrade the orbit prediction accuracy (particularly periapsis timing) in support of orbiter maneuvers and science experiments. The relative merit of each new gravity model was determined by how accurately the observed times of periapses could be predicted. Early successes proved that timing errors could be held to within ±1.0 s after ten revolutions of prediction, which compares to ±10.0 s had no improvements been made. [1]

In the current analysis, the Viking data were reduced to obtain a model of degree and order six in the spherical harmonic expansion of the potential. Since this model is restricted to low-degree terms, the technique used here takes advantage of the orbit-to-orbit variations of the orbital elements, particularly the mean period. This method precludes modeling short-period variations near periapsis which usually require higher-degree terms in the potential. The model discussed herein was derived from an ensemble of models each estimated from Doppler data obtained over two to four revolutions during various synchronous and asynchronous phases of the Viking mission. A priori Mariner-9 fields were also included in the ensemble to apply the requisite constraint on the gravity in the south.

Comparisons between our first Mars gravity models and the models derived from Mariner-9 tracking data revealed favorable agreement of geoid (areoid) heights in the north and over the Tharsis ridge, but showed marked disagreements in the south. Disagreements in the south can be explained by the fact that Viking periapses were confined to the north. In fact, the Viking orbiters best sense the gravity field in the vicinity of 25°N to 50°N, whereas Mariner-9 models are most valid in the vicinity of 25°S. This indicated that, in order to retain the inherent local accuracy of the constituent fields while obtaining a model which was valid globally, the proper weighting of a priori Mariner-9 data was required.

Similar gravity analyses using data from the low-altitude phases (300 km for Viking-1 and 800 km for Viking-2) indicate that sixth-degree and order models adequately recover local gravity perturbations; however, these models in ensemble degrade the gravity field globally. In other words, due to suboptimal filtering, a sixth-degree and order field fit the local orbit perturbations at the expense of the global field. Therefore, we believe that future data reduction incorporating the low-altitude Viking orbits will require higher degree and order models. Further study is warranted since preliminary low-altitude results suggest interesting detail in the equipotential surface.

The following sections will first describe the techniques used by the Viking Navigation Team to improve the gravity model, and then our current gravity field estimate will be presented.

Analysis and Method

Orbit-to-orbit variations are quite obvious in the evolution of the anomalistic period for each Viking orbiter. Conceptually, the period experiences a change (ΔP) upon each periapsis crossing due to gravitational perturbations. As a result, the predicted times of periapsis evolve as

$$t_p \sim \tfrac{1}{2} n(n+1)\Delta P \tag{1}$$

for a synchronous orbiter, where n is the number of predicted periapses and ΔP is essentially constant. This can be seen by using Kaula's expression for the disturbing function[2] to obtain

$$\dot{P} = 6\pi \sum_{lm} \left(\frac{R}{a}\right)^l J_{lm} \sum_{pq} F_{lmp}(I)\, G_{lpq}(e) \left\{ \begin{matrix} -\sin \\ \cos \end{matrix} \right\} \begin{matrix} {\scriptstyle l-m \text{ even}} \\ \phi_{lmpq} \\ {\scriptstyle l-m \text{ odd}} \end{matrix} \tag{2}$$

$$\phi_{lmpq} = (l-2p)\omega + (l-2p+q)M + m(\beta - \lambda_{lm})$$

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where

R	= planet radius
J_{lm}	= harmonic coefficient
F_{lmp}	= inclination function
G_{lpq}	= eccentricity expansion
a	= semimajor axis
I	= inclination
e	= eccentricity
ω	= argument of perifocus
M	= mean anomaly
λ_{lm}	= longitude of harmonic

The graphic node (β) is defined as $\beta = \Omega - \theta$ where θ is the hour angle and Ω is the longitude of the ascending node (see Fig. 1). Under the condition of resonance:

$$(l-2p+q)\dot{M}+m\dot{\beta}=0 \tag{3}$$

so terms corresponding to $l-2p+q=m(m\neq 0)$ result in $M+\beta \approx$ constant and $\dot{P} \approx$ constant. It is convenient to evaluate $M+\beta$ at the time of periapsis so all future reference to β will imply the graphic node at periapsis.

On the other hand, for an asynchronous orbiter, ΔP may alter sign and magnitude for different periapsis crossings so that the time of periapsis does not grow geometrically but instead varies as

$$t_p \sim \sum_{i=0}^{n-1}(n-i)\Delta P_i \tag{4}$$

Error analyses as well as real-time experiences[1] show that ΔP_i can be recovered to within 0.02 s once estimated over two revolutions (i.e. one periapsis crossing) of Viking orbiter data. This can be compared to a ΔP_i error on the order of a second for various Mariner-9 models; thus, significant improvement in the knowledge of the gravity field near the subperiapsis point can be realized. Furthermore, these analyses show that gravity modeling errors primarily affect the propagation of errors in orbit event times [such as periapsis time, see Eqs. (1) and (4)] while orientation errors due to the orbit determination process are propagated virtually unchanged. Thus, the merit of any gravity model can be judged by how accurately it predicts times of periapses, or equivalently, how accurately it predicts ΔP_i. This concept is the basis for the strategies and techniques adopted here.

The procedure used to update the Mars gravity model consisted of first performing batch least-squares fits over two or more revolutions of the Doppler data, while estimating satellite state and sixth-degree and order gravity field parameters, and then combining these localized gravity estimates in a linear sense to form an ensemble model. In order to recover only medium period (~24 h) variations with individual short arc fits, data are deleted within 1 h of each periapsis, and an a priori Mariner-9[3] field with covariance is used to better condition the filter. This strategy permits fair recovery of the local field while substantially reducing the error in ΔP_i.

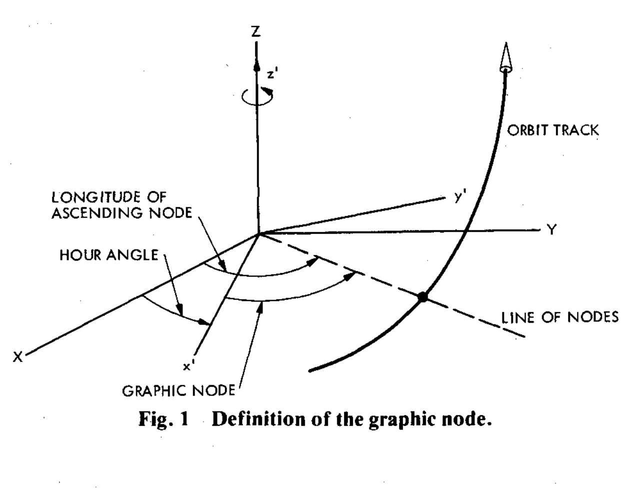

Fig. 1 Definition of the graphic node.

Most Viking orbits have periods different from Mars' rotational period (24.6228 h), so they experience "walks," i.e. planetary circulation of the subperiapse point. As a result, new information is gained on each periapsis crossing, thus permitting gravity sensing over a broad range of longitudes. Using the technique discussed below, an ensemble gravity model can be formed which retains the information contained in the individual short arc fields determined over a variety of longitudes.

For the current analysis, the a priori Mariner-9 field[3] (C_0) with covariance (Γ_0) is removed from a set of short arc (two to four revolutions) estimates obtained from synchronous and walk phases of the Viking mission. This is done in order to extract the gravity information inherent in the data by removing the effect of a priori knowledge. Each short arc solution and associated covariance will be referred to as

$$\hat{C}_i' = \hat{C}_i - \Gamma_i'\Gamma_0^{-1}(C_0-\hat{C}_i) \tag{5}$$

as data and

$$\Gamma_i'^{-1} = \Gamma_i^{-1} - \Gamma_0^{-1} \tag{6}$$

as the data weight applied to a least-squares estimator

$$\hat{C} = \left(\sum_i \Gamma_i'^{-1} + \tilde{\Gamma}_0^{-1}\right)^{-1}\left[\sum_i \Gamma_i'^{-1}\hat{C}_i'\tilde{\Gamma}_0^{-1}\hat{C}_0\right] \tag{7}$$

Where $\hat{C}$ is to be taken as the best estimate of the field given the ensemble of all available data. Note that $\hat{C}_i'$ and Γ_i' are the short arc estimates and covariances obtained if no a priori knowledge is assumed. A provision is made for applying a priori ($\tilde{C}_0, \tilde{\Gamma}_i$) to the ensemble to effectively constrain the resulting field globally. The choice of this a priori field (or fields) and its relative weighting in the filter will be discussed in the section on Data Reduction. It must be pointed out that long-term gravity effects are not included in this estimator since the states at the beginning of each short arc are not connected dynamically (i.e. they are determined independently). It has been shown that this is not a serious shortcoming, as was previously demonstrated by the successful application of short arc techniques to Viking[4] and Mariner-9[5,6] data.

Data Reduction

Figure 2 shows all orbits (data spans) over which short arc gravity estimates were obtained. Included in the table is the period, inclination, periapsis altitude, latitude, and longitude appropriate to those phases of the mission. Data limitations are primarily due to 1) high solar-induced noise from mid-November 1976 to late January 1977 during solar conjunction, 2) lack of contiguous orbits with continuous coverage, and 3) lack of new information in the data. The data fit covers a band of northern latitudes with nearly global coverage in longitude owing to the walk phases of VO-1 and VO-2. The corresponding subperiapse points for these data spans are included in Fig. 3. The concentration of data over the Chryse basin (longitudes 20° to 40° W) can be attributed to the early synchronous phases of VO-1 (periapses numbers 2 through 46).

Including all of these data in ensemble may completely overwhelm an a priori Mariner-9 field supplied to the estimator, Eq. (7). However, a proper choice of a priori can diminish this effect. In order to determine the proper weighting of an a priori field in the ensemble, three methods were devised. First, a correction factor for the computed covariance of the nominal field[3] was determined based on the

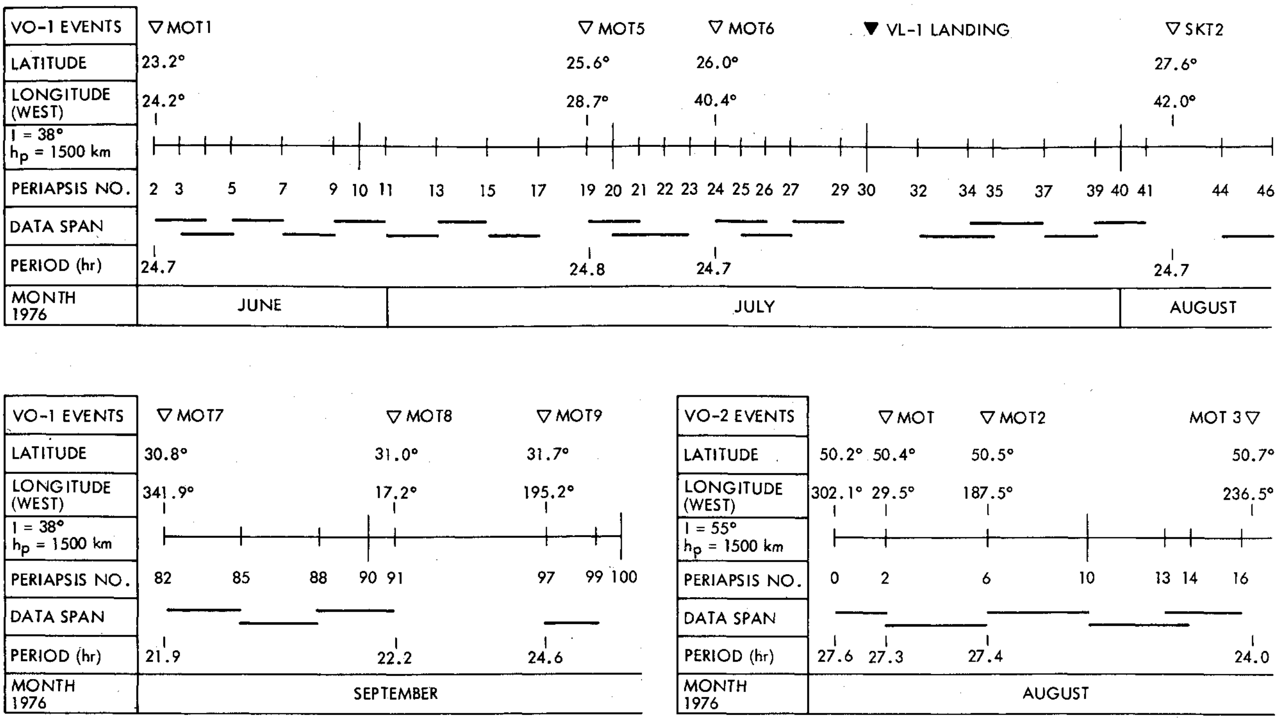

Fig. 2 Viking-1 and Viking-2 data spans included in gravity estimate: MOT-Mars Orbiter Trim, SKT-Station Keeping Trim.

Table 1 Spherical harmonic coefficients $\times 10^7$ based on Viking-1, Viking-2, and Mariner-9 data

Not normalized						Normalized					
Coefficient		Sigma	Coefficient		Sigma	Coefficient		Sigma	Coefficient		Sigma
J2	19590.4068	1.3870				J2	8761.1	.6			
J3	329.3520	1.6908				J3	124.5	.6			
J4	−209.0859	2.9036				J4	−69.7	1.0			
J5	78.1660	4.6366				J5	23.6	1.4			
J6	−43.4028	4.4453				J6	−12.0	1.2			
C21	.0000	.0000	S21	.0000	.0000	C21	.0	.0	S21	.0	.0
C22	−552.2794	.1178	S22	314.8175	.1381	C22	−855.6	.2	S22	487.7	.2
C31	50.6698	.3052	S31	269.2670	.3279	C31	46.9	.3	S31	249.3	.3
C32	−57.5203	.1457	S32	28.4776	.1478	C32	−168.4	.4	S32	83.4	.4
C33	49.3225	.0343	S33	34.7968	.0315	C33	353.7	.2	S33	249.5	.2
C41	42.9437	.6785	S41	53.2863	.6204	C41	45.3	.7	S41	56.2	.7
C42	−4.0877	.1400	S42	−21.1722	.1464	C42	−18.3	.6	S42	−94.7	.7
C43	2.9380	.0398	S43	1.4011	.0394	C43	49.2	.7	S43	23.4	.7
C44	−.3267	.0191	S44	−2.6250	.0189	C44	−15.5	.9	S44	−124.2	.9
C51	−10.0990	.8964	S51	1.7392	.8840	C51	−11.8	1.0	S51	2.0	1.0
C52	−8.7924	.1507	S52	−1.2402	.1381	C52	−54.3	.9	S52	−7.7	.9
C53	.6404	.0261	S53	.4700	.0283	C53	19.4	.8	S53	14.2	.9
C54	−.4234	.0058	S54	−.0828	.0054	C54	−54.4	.7	S54	−10.6	.7
C55	−.0904	.0017	S55	.1476	.0019	C55	−36.7	.7	S55	59.9	.8
C61	61.4269	1.1086	S61	−14.3979	1.2077	C61	78.1	1.4	S61	−18.3	1.5
C62	1.5655	.1187	S62	1.1896	.1228	C62	12.6	1.0	S62	9.6	1.0
C63	.9745	.0241	S63	.9316	.0297	C63	47.0	1.2	S63	44.9	1.4
C64	.1801	.0022	S64	.0124	.0011	C64	47.6	.6	S64	3.3	.3
C65	.0212	.0006	S65	.0273	.0007	C65	26.3	.8	S65	33.9	.9
C66	.0016	.0001	S66	.0030	.0001	C66	6.9	.4	S66	12.7	.5

difference between its expected errors and its demonstrated prediction capability. Next, if one assumes that the Mariner-9 data is just as valid in the south as the Viking data is in the north, then two further constraints on the a priori weighting are possible. One of these was to qualitatively determine the relative a priori weighting by comparing the areoids obtained from the a priori and ensemble fields. Large differences in areoid heights over the Mariner-9 sub-periapse region would then indicate that a stronger weighting of Mariner-9 should be made. The third and final test was the comparison of the degree uncertainties in the two sets of harmonic coefficients. This was done by summing the variances for each degree harmonic along the diagonal of the covariances obtained from Mariner-9 data and Viking data respectively. For nearly equal weighting, the ratios of these spectral sums should be close to unity.

In the present analysis, all three tests were used to insure proper weighting of the a priori field. With reference to the first test, early investigations of the nominal gravity field[3] revealed that the observed timing performance was three times better than expected. This led to a preliminary Viking gravity model obtained by combining fields determined over data arcs shown in Fig. 2 with the nominal Mariner-9 field weighted by a factor of nine. This model was compared to

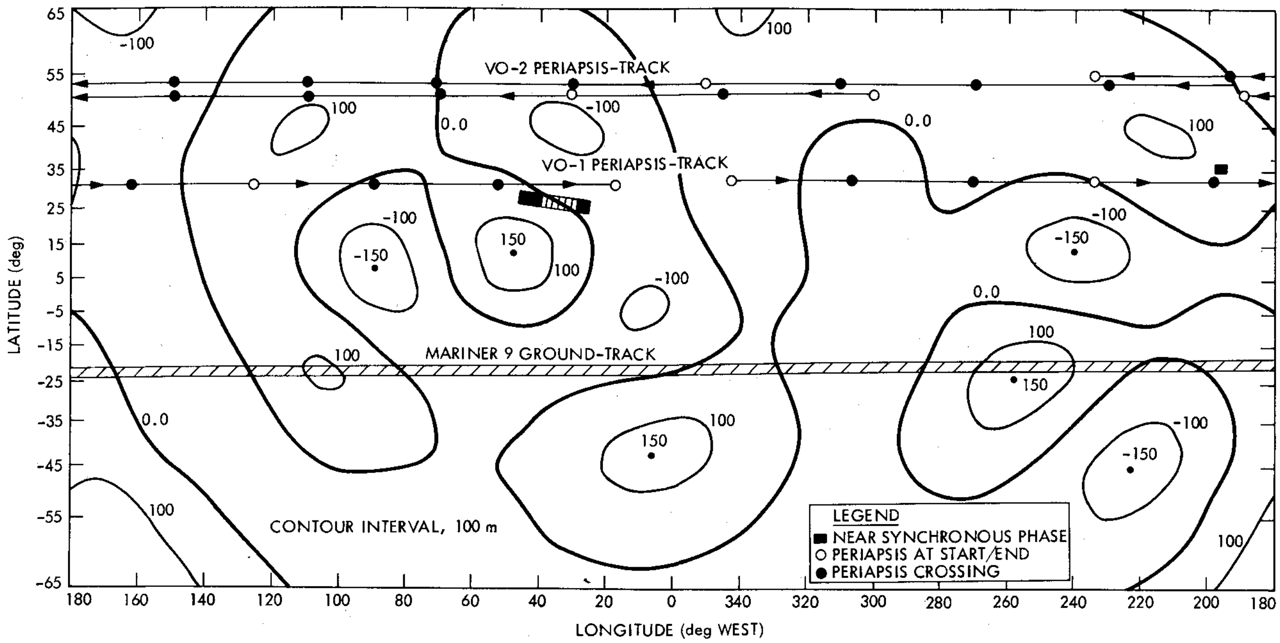

Fig. 3 Differences in equipotential heights for model VM6×6/JPL minus the Mariner-9 nominal with sub-periapse points from Fig. 2 superimposed. The Mariner-9 ground track is the region of subperiapses only.

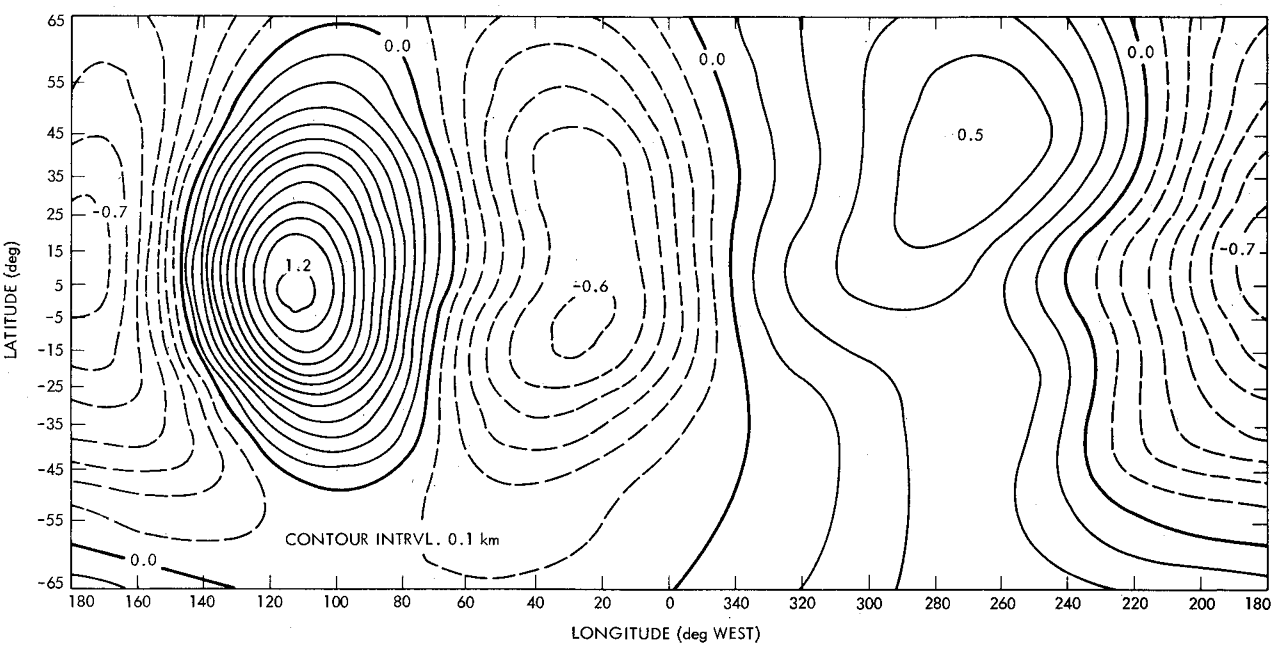

Fig. 4 Areoid heights based on model VM6×6/JPL.

certain independently determined Mariner-9 models, and its ability to predict the change in period over the first VO-1 synchronous phase was superior.[1] Despite the successful performance of this model early in the mission, it was found to be globally distorted as evidenced by large discrepencies (≈ 350 m) in the areoid in the south.

Extending the Viking gravity model to include data from the low-altitude phases (300 km for Viking-1 and 800 km for Viking-2), using the methods just discussed, revealed interesting detail in the gravity field but presented a number of problems. Although this data could be reduced to yield sixth-degree and order gravity harmonics which recovered the change in orbital period (ΔP) as before, these fields could not be combined in ensemble without causing distortions of up to 300 m in the areoid. The reason for this is that sixth-degree and order harmonic coefficients cannot simultaneously describe the global and local undulations of the field at the lower altitudes. In fact, a field of degree and order 36 would be required to properly resolve features as small as 300 km, i.e. features on the order of the spacecraft altitude. However, the areoids produced from this data, although distorted globally, showed increased detail in the sub-periapse regions. Especially noteworthy were detailed features in the equipotential surface which correlated well with the topographical regions of Valles Marineris and Elysium Mons.

Because of the increased global distortion, all further efforts were directed toward improving a sixth degree and order global model using the 1500 km altitude Viking data with Mariner-9 a priori.

Gravity Model

In order to reduce discrepancies in the southern hemisphere, a different Mariner-9 model[6] was introduced into the ensemble outlined in Fig. 2. The previously discussed spectral analysis of the degree variances on the covariances indicated that this field has essentially equal weight with the ensemble Viking field. The resulting set of harmonic coefficiences which represent the current Viking gravity model is given in Table 1 along with their associated uncertainties. Navigation experience with this field, called Model VM6×6/JPL, indicates that these formal uncertainties may be over-optimistic by more than a factor of ten.

The equipotential surface (areoid) based on model VM6×6/JPL is presented in Fig. 4. This surface is referred to a spheroid of flattening 1/191.755 and mean radius 3393.4 km with undulations represented at 100 m intervals. The dominant characteristic of all Martian gravity models is the equatorial ellipticity, evidenced by alternating positive and negative longitudinal quadrants, arising from the sectoral of degree and order two. This produces the 1.2 km high in Tharsis (115°W, 0°N) flanked by a 0.7 km low to the west and a 0.6 km low to the east. A dichotomy exists in the quadrant centered at 280°W where the northern half is dominated by a 0.5 km high and the southern half by a 0.2 km saddle over the Hallas Basin (290°W, 40°S). Of further interest is the steep gradient along the eastern slope of Tharsis, descending to the Chryse Basin (30°W, 10°N), which is preferentially aligned with Valles Marineris (approximately 10°S). The strong correlation between gravity and topography in this region has been evident in previously determined fields, but an even stronger correlation was noted during our low-altitude investigations.

Figure 3 represents the difference in areoid heights between model VM6×6/JPL and the nominal Mariner-9 field.[3] Both areoids agree to within ±150 m over the entire planet. As pointed out by the Radio Science Team,[4] Viking data has substantiated fields determined from Mariner-9 to within 150 m variations in the areoid, even in the Viking sub-periapse regions. Even though model VM6×6/JPL has Mariner-9 data included via a priori, analysis has shown that the areoid presented in Fig. 4 does not differ significantly from an areoid determined solely from Viking data in the Viking sub-periapse regions. In other words, the a priori Mariner-9 field does not alter but actually compliments the gravity field observed by Viking in the north.

Conclusions

Navigation results show that the linear combination of short arc gravity estimates in an acceptable technique for obtaining gravity models from multiple data arcs without iteration. In fact, an ensemble field composed of Viking data and Mariner-9 a priori retains the inherent local accuracy of its constituent fields. At the same time, the model can be made to be valid globally by careful weighting of a priori Mariner-9 data. The sixth-degree and order model (VM6×6/JPL) presented here reduces the error in ΔP_i by more than an order of magnitude during the high altitude (1500 km) phases of the Viking mission. The resulting areoid deviates by no more than 150 m from the areoid produced by the nominal Mariner-9 field.[3] This not only suggests that the model is valid globally, but also indicates that the northern hemisphere is well represented by Mariner-9 models. This is evident from the navigation results and the equipotential surfaces (Figs. 3,4).

When data taken during the low-altitude (800 km and 300 km) phases of the mission was reduced, it was found that a sixth-degree and order model failed to produce an acceptable global field. Preliminary analyses show that high degree and order terms will be required to properly represent the Martian gravity field during these low-altitude phases of the mission.

The gravity reduction technique presented here does not attempt to model short-period ($\ll$24 h) or long-period ($\gg$24 h) orbital element variations. This was done in order to facilitate quick recovery of the gravity field by minimizing computation requirements. In fact, by recovering only those orbit perturbations commensurate with the orbital period, this method avoids both the additional computer storage required for a higher degree and order model and the longer integration time required to filter long-period variations. The results presented here show that ignoring these variations did not prevent improvement of the Viking navigation accuracy. In other words, the strength of this method rests in its ability to produce an accurate global gravity model with computational efficiency.

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